



REGLAS FUNDAMENTALES DE DIFERENCIACION

POR

CARLOS WARGNY

(Continuacion)

$$\begin{aligned}
 7. \quad y &= \cos (a-bx) \therefore dy = -\text{sen } (a-bx) d (a-bx) \\
 &= -\text{sen } (a-bx) \cdot -b dx = b \text{ sen } (a-bx) dx.
 \end{aligned}$$

$$\begin{aligned}
 8. \quad d \cos \sqrt{ax^2+bx+c} &= -\text{sen } \sqrt{ax^2+bx+c} d(ax^2+bx+c)^{\frac{1}{2}} \\
 &= -\text{sen } \sqrt{ax^2+bx+c} \cdot \frac{2ax+b}{2\sqrt{ax^2+bx+c}} dx
 \end{aligned}$$

$$\begin{aligned}
 9. \quad d \cos nx \cos mx &= \cos nxd \cos mx + \cos mxd \cos nx \\
 &= -\cos nx \text{ sen } mxdm - \cos mx \text{ sen } nxdn
 \end{aligned}$$

$$=-m \cos nx \operatorname{sen} mx - n \cos mx \operatorname{sen} nxdx.$$

$$270. d \cos^n x = n \cos^{n-1} x d \cos x = -n \cos^{n-1} x \operatorname{sen} x dx$$

$$1. d \operatorname{sen}^m x \cos^n x = x (\operatorname{sen}^m x \cdot n \cos^{n-1} x \cdot$$

$$- \operatorname{sen} x + \cos^n x \cdot m \operatorname{sen}^{m-1} x \cos x) dx$$

$$= (-\operatorname{sen}^{m+1} x \cos^{n-1} x + D \cos^{n+1} x \operatorname{sen}^{m-1} x) dx$$

$$2. y = L \cos x \therefore dy = \frac{d \cos x}{\cos x} = \frac{-\operatorname{sen} x}{\cos x} dx = -\operatorname{tg} x dx$$

$$3. y = (\cos x)^{\operatorname{sen} x} \therefore L y = \operatorname{sen} x L \cos x$$

$$\frac{dy}{y} = \operatorname{sen} x \cdot -\operatorname{tg} x dx + L \cos x \cdot \cos x dx$$

$$dy = \cos x^{\operatorname{sen} x} (-\operatorname{sen} x \operatorname{tg} x + L \cos x^{\cos x}) dx$$

$$4. y = \cos (x^{\operatorname{sen} x}) \therefore dy = -\operatorname{sen} (x^{\operatorname{sen} x}) d (x^{\operatorname{sen} x}).$$

Hacemos $u = x^{\operatorname{sen} x}$ y aplicamos L :

$$L u = \operatorname{sen} x L x \therefore \frac{du}{u} = \left(\frac{\operatorname{sen} x}{x} + \cos x L x \right) dx$$

$$du = x^{\operatorname{sen} x} \left(\frac{\operatorname{sen} x}{x} + L x^{\cos x} \right) dx$$

$$\therefore dy = -\operatorname{sen} (x^{\operatorname{sen} x}) x^{\operatorname{sen} x} \left(\frac{\operatorname{sen} x}{x} + L x^{\cos x} \right) dx$$

$$175. \quad y = \cos \operatorname{sen} (1-x). \quad dy = -\operatorname{sen} \operatorname{sen} (1-x)$$

$$d \operatorname{sen} (1-x) = -\operatorname{sen}_2 (1-x) \cos (1-x) d(1-x)$$

$$= \operatorname{sen}_2 (1-x) \cos (1-x) dx$$

Diferencial de la tangente.—Sea $y = \operatorname{tg} u$.

$$dy = d \operatorname{tg} u = d \frac{\operatorname{sen} u}{\cos u} = \frac{\cos u d \operatorname{sen} u - \operatorname{sen} u d \cos u}{\cos^2 u}$$

$$= \frac{\cos^2 u + \operatorname{sen}^2 u}{\cos^2 u} du = \frac{1}{\cos^2 u} du = \sec^2 u du$$

$$\therefore d(\operatorname{tg} u) = \sec^2 u du \quad (14)$$

La diferencial de la tangente es igual al cuadrado de la secante por la diferencial del arco.

$$\text{Ejercicio 276 } y = \operatorname{tg} (ax^2 + bx + c) \therefore dy = \sec^2 (ax^2 + bx + c)$$

$$d(ax^2 + bx + c) = (2ax + b) \sec^2 (ax^2 + bx + c) dx.$$

$$7. \quad d \operatorname{tg}^n x^m = n \operatorname{tg}^{n-1} x^m d \operatorname{tg} x^m = n \operatorname{tg}^{n-1} x^m \sec^2 x^m dx^m = \\ m n x^{m-1} \sec^2 x^m \operatorname{tg}^{n-1} x^m dx.$$

DIFERENCIAL DE LA COTANGENTE. $y = \cot u$

$$\begin{aligned} d \cot u &= d \frac{1}{\operatorname{tg} u} = - \frac{d \operatorname{tg} u}{\operatorname{tg}^2 u} = - \frac{\sec^2 u}{\operatorname{tg}^2 u} = - \frac{1}{\cos^2 u} \frac{\cos^2 u}{\sin^2 u} \\ &= - \frac{1}{\sin^2 u} \therefore d(\cot u) = - \operatorname{cosec}^2 u. \end{aligned}$$

DIFERENCIAL DE LA SECANTE $y = \sec u$.

$$d \sec u = d \frac{1}{\cos u} = \frac{d \cos u}{\cos^2 u} = \frac{\sin u}{\cos^2 u} du = \operatorname{tg} u \sec u du$$

$$\therefore d(\sec u) = \sec u \operatorname{tg} u du.$$

DIFERENCIAL DE LA COSECANTE. $y = \operatorname{cosec} u$

$$(81) \quad d \operatorname{cosec} u = d \frac{1}{\sin u} = - \frac{d \sin u}{\sin^2 u} = - \frac{\cos u}{\sin^2 u} du$$

$$\therefore d(\operatorname{cosec} u) = - \cot u \operatorname{cosec} u du$$

DIFERENCIAL DEL SENO VERSO. $y = \operatorname{vers} u$

$$d \operatorname{vers} u = d(1 - \cos u) = \sin u du$$

$$\therefore d(\operatorname{vers} u) = \sin u du.$$

DIFERENCIAL DEL COSENO VERSO. $y = \text{covers } u$

$$d \text{ covers } u = d(1 - \sin u) = -\cos u du$$

$$\therefore d(\text{covers } u) = -\cos u du$$

Ejer. 278. $y = \frac{\sin^m x}{\cos^n x}$. (Williamson, 27)

$$L y = m L \sin x - n L \cos x$$

$$\frac{d y}{y} = m \frac{d \sin x}{\sin x} - n \frac{d \cos x}{\cos x}$$

$$dy = \frac{\sin^{m-1} x}{\cos^{n-1} x} (m \cos^2 x + n \sin^2 x) dx$$

9. $d \sin L \operatorname{tg} x = \cos L \operatorname{tg} x d L \operatorname{tg} x$

$$= \cos L \operatorname{tg} x \frac{d \operatorname{tg} x}{\operatorname{tg} x} = \cos L \operatorname{tg} x \frac{\sec^2 x}{\operatorname{tg} x} dx$$

280. $y = \frac{\sec \operatorname{cosec} x}{\operatorname{tg} \cot x}$

$$\frac{dy}{dx} = \frac{\operatorname{tg} \cot x \operatorname{tg} \operatorname{cosec} x \sec \operatorname{cosec} x + \sec \operatorname{cosec} x \sec^2 \cot x \cot x \operatorname{cosec} x}{\operatorname{tg}^2 x \cot x}$$

* II. MÉTODO.—En Algebra Superior se demuestran las ecuaciones de Euler, en las que las líneas trigonométricas, mediante el signo $i=\sqrt{-1}$, toman la forma exponencial:

$$\text{sen } x = \frac{e^{ix} - e^{-ix}}{2i}, \quad \text{cos } x = \frac{e^{ix} + e^{-ix}}{2}.$$

Diferenciamos la primera y resulta la segunda como derivada:

$$\begin{aligned} d \text{ sen } x &= d \frac{e^{ix} - e^{-ix}}{2i} = \frac{1}{2i} (d e^{ix} - d e^{-ix}) \\ &= \frac{1}{2i} [e^{ix} dx - e^{-ix} d(-ix)] = \frac{1}{2i} (e^{ix} i + e^{-ix} i) dx \\ &= \frac{1}{2} (e^{ix} + e^{-ix}) dx = \text{cos } x dx \end{aligned}$$

$$\therefore d \text{ sen } x = \text{cos } x dx.$$

Para encontrar las demás diferenciales trigonométricas basta reducir las líneas á sen y cos: $\text{cos } x = \text{sen } (90^\circ x)$, $\text{tg } x = \frac{\text{sen } x}{\text{cos } x}$, etc.

29. Funciones circulares.—A). DEFINICIONES. Para despejar á y de la función trigonométrica

$$u = \text{sen } y,$$

en la que y es el arco correspondiente al seno u , se conviene en escribir

$$y = \text{arc sen } u.$$

y se lee «arco cuyo seno es u » ó, mejor, «arco seno u ».

Tal es la FUNCIÓN CIRCULAR SENO.

De los demás funciones trigonométricas,

$$u = \cos y, u = \text{tg } y, u = \sec y, \text{ etc.};$$

salen las funciones circulares respectivas,

$$y = \text{arc cos } u, y = \text{arc tg } u, y = \text{arc sec } u, \text{ etc.}$$

Según esto, las funciones circulares expresan el arco en términos de las líneas trigonométricas; y son, por tanto, inversas de las funciones trigonométricas.

B). VALOR NUMÉRICO DE LAS FUNCIONES CIRCULARES. — El valor conocido $\text{sen } 30^\circ = \frac{1}{2}$ nos dá: $30^\circ = \text{arc sen } \frac{1}{2}$; y como $360^\circ = 2\pi$ r o $180^\circ = \pi$, tendremos:

$$\frac{1}{6} \pi = \text{arc sen } 0,5 \quad \dots \quad \pi = 6 \text{ arc sen } 0,5$$

Para $\text{tg } 45^\circ = 1$, encontramos, $45^\circ = \frac{\pi}{4} = \text{arc tg } 1$; y

$$\text{para } \cos 30^\circ = \frac{1}{2} \sqrt{3}, \quad \frac{1}{6} \pi = \arccos \frac{1}{2} \sqrt{3}.$$

Luego,

$$\pi = 6 \arcsin 0,5 = 4 \arctg 1 = 6 \arccos \frac{1}{2} \sqrt{3}.$$

C). RELACIONES CIRCULARES.

a). Sea $y = \arcsin u \therefore u = \sin y$,

$$1 - u^2 = 1 - \sin^2 y = \cos^2 y \therefore \cos y = \sqrt{1 - u^2}, \quad y = \arccos \sqrt{1 - u^2}.$$

Luego,

$$\arcsin u = \arccos \sqrt{1 - u^2}$$

Si se quiere expresar $\arcsin u$ en función de $\arctg u$, escribimos:

$$\operatorname{tg} y = \frac{\sin y}{\cos y} = \frac{\sin y}{\sqrt{1 - \sin^2 y}} = \frac{1}{\sqrt{1 - u^2}} \therefore y = \arctg \frac{u}{\sqrt{1 - u^2}}$$

Luego,

$$\arcsin u = \arctg \frac{u}{\sqrt{1 - u^2}}$$

Del mismo modo se encuentran las relaciones:

$$\operatorname{arc} \operatorname{sen} u = \operatorname{arc} \cot \frac{\sqrt{1-u^2}}{u} = \operatorname{arc} \sec \frac{1}{\sqrt{1-u^2}} = \operatorname{arc} \operatorname{cosec} \frac{1}{u}$$

8). Calculemos, ahora, la suma $\operatorname{arc} \operatorname{sen} a + \operatorname{arc} \operatorname{sen} b$.

$$\text{Sea } \operatorname{arc} \operatorname{sen} a = u \quad \therefore a = \operatorname{sen} u, \quad \cos u = \sqrt{1-a^2},$$

$$\operatorname{arc} \operatorname{sen} b = v \quad \therefore b = \operatorname{sen} v, \quad \cos v = \sqrt{1-b^2}.$$

$$\text{Sabemos que } \operatorname{sen} (u+v) = \operatorname{sen} u \cos v + \operatorname{sen} v \cos u;$$

$$\text{luego,} \quad u+v = \operatorname{arc} \operatorname{sen} (\operatorname{sen} u \cos v + \operatorname{sen} v \cos u)$$

$$= \operatorname{arc} \operatorname{sen} (a \sqrt{1-b^2} + b \sqrt{1-a^2})$$

c). Calculemos, además, $\operatorname{arc} \operatorname{tg} a + \operatorname{arc} \operatorname{tg} b$.

$$\text{Sea } \operatorname{arc} \operatorname{tg} a = u \quad \therefore a = \operatorname{tg} u; \quad \operatorname{arc} \operatorname{tg} b = v \quad \therefore b = \operatorname{tg} v.$$

$$\operatorname{tg} (u+v) = \frac{\operatorname{tg} u + \operatorname{tg} v}{1 - \operatorname{tg} u \operatorname{tg} v} \quad \therefore u+v = \operatorname{arc} \operatorname{tg} \frac{\operatorname{tg} u + \operatorname{tg} v}{1 - \operatorname{tg} u \operatorname{tg} v}$$

$$\text{o bien,} \quad \operatorname{arctg} a + \operatorname{arctg} b = \operatorname{arctg} \frac{a+b}{1-ab}.$$

d). Hacemos $a = b$ en las fórmulas anteriores:

$$2 \operatorname{arc} \operatorname{sen} a = \operatorname{arc} \operatorname{sen} 2 a \sqrt{1-a^2},$$

$$2 \operatorname{arc} \operatorname{tg} a = \operatorname{arc} \operatorname{sen} \frac{2a}{1-a^2}. \quad (\text{Greenhill, 27})$$

DIFERENCIAL DE ARCO SENO.— $y = \operatorname{arc} \operatorname{sen} u$.

$$u = \operatorname{sen} y, du = \cos y \, dy, dy = \frac{du}{\cos y}.$$

Para eliminar a y del segundo miembro, acudimos a igualdad $\operatorname{sen} y = u \therefore 1 - \operatorname{sen}^2 y = 1 - u^2 = \cos^2 y$

$$\therefore \cos y = \sqrt{1-u^2}$$

$$\therefore d(\operatorname{arc} \operatorname{sen} u) = \frac{du}{\sqrt{1-u^2}} \quad (15)$$

La diferencial de arco seno u es igual a la diferencial de u dividida por la raíz cuadrada de la unidad disminuida en el cuadrado de u .

$$\text{Ej. 281. } d \operatorname{arc} \operatorname{sen} (ax+b) = \frac{d(ax+b)}{\sqrt{1-(ax+b)^2}} = \frac{adx}{\sqrt{1-(ax+b)^2}}$$

$$2. \, d \operatorname{arc} \operatorname{sen} \frac{1}{x} = \frac{d \frac{1}{x}}{\sqrt{1-\left(\frac{1}{x}\right)^2}} = \frac{-\frac{1}{x^2}}{\sqrt{1-\frac{1}{x^2}}} dx = -\frac{dx}{x\sqrt{x^2-1}}.$$

$$3. d (\text{arc sen } \sqrt{x})^n = d \cdot \text{arc}^n \text{ sen } \sqrt{x} = n \text{ arc}^{n-1} \text{ sen } \sqrt{x}$$

$$d \text{ arc sen } \sqrt{x} = n \text{ arc}^{n-1} \text{ sen } \sqrt{x} \frac{dx}{2\sqrt{x-x^2}} =$$

$$n \text{ arc}^{n-1} \text{ sen } \sqrt{x} \cdot \frac{dx}{2\sqrt{x-x^2}}.$$

$$4. d L \text{ arc sen } x = \frac{d \text{ arc sen } x}{\text{arc sen } x} = \frac{dx}{\text{arc sen } x \cdot \sqrt{1-x^2}}.$$

$$5. y = x^{\text{arc sen } x}. \quad L y = \text{arc sen } x L x$$

$$\frac{dy}{y} = \text{arc sen } x \frac{dx}{x} + L x \frac{dx}{\sqrt{1-x^2}}$$

$$\therefore dy = x^{\text{arc sen } x} \left(\frac{\text{arc sen } x}{x} + \frac{L x}{\sqrt{1-x^2}} \right) dx$$

DIFERENCIAL DE ARCO COSENO. $y = \text{arc cos } u$.

$$u = \cos y, du = -\text{sen } y dy = -\sqrt{1-u^2} dy$$

$$\therefore d (\text{arc cos } u) = -\frac{du}{\sqrt{1-u^2}}$$

La diferencial de $\text{arc cos } u$, es igual a la de $\text{arc sen } u$, con signo o menos.

$$\therefore \pm \frac{du}{\sqrt{1-u^2}} = d \text{ arc sen } u \text{ o } d \text{ arc cos } u.$$

DIFERENCIAL DE ARCO TANGENTE. $y = \text{arc tg } u$.

$$u = \text{tg } y, du = \sec^2 y dy = (1 + \text{tg}^2 y) dy = (1 + u^2) dy:$$

$$\therefore d(\text{arc tg } u) = \frac{du}{1+u^2} \quad (16)$$

La diferencial de $\text{arc tg } u$ es igual á du dividida por la unidad aumentada en el cuadrado de u .

$$\text{Ej. 286. } d \text{ arc tg } (a + bx + cx^2) = \frac{d(a + bx + cx^2)}{1 + (a + bx + cx^2)^2}$$

$$= \frac{b + 2cx}{1 + (a + bx + cx^2)^2} dx$$

$$7. D \text{ arc tg } \frac{3x}{1+x^2} = D \frac{3x}{1+y^2} : \left[1 + \left(\frac{3x}{1+x^2} \right)^2 \right]$$

$$= \frac{(1+x^2) 3 - 3x \cdot 2x}{(1+x^2)^2} \cdot \frac{(1+x^2)^2}{9x^2} = \frac{3-3x^2}{9x^2(1+x^2)} = \frac{1-x^2}{3x^2(1+x^2)^2}$$

$$8. y = \text{arc tg } \sqrt{\frac{1-\cos x}{1+\cos x}}. \quad (\text{Bowser, 57})$$

$$\sqrt{\frac{1-\cos x}{1+\cos x}} = \sqrt{\frac{(1-\cos x)^2}{1-\cos^2 x}} = \frac{1-\cos x}{\sin x} = \frac{2 \sin^2 \frac{1}{2} x}{2 \sin \frac{1}{2} x \cos \frac{1}{2} x}$$

$$= \operatorname{tg} \frac{1}{2} x \therefore y = \operatorname{arc} \operatorname{tg} \operatorname{tg} \frac{1}{2} x \text{ ó } \operatorname{tg} \frac{1}{2} x = \operatorname{tg} y$$

$$\text{ó } y = \frac{1}{2} x \therefore dy = \frac{1}{2} dx.$$

$$9. \quad y = L \operatorname{arctg} u. \quad dy = \frac{d \operatorname{arctg} u}{\operatorname{arc} \operatorname{tg} u}$$

$$= \frac{du}{(1+u^2) \operatorname{arctg} u}.$$

$$290. \quad y = (\operatorname{sen} x) \operatorname{arctg} u. \quad L y = \operatorname{arctg} u \quad L \operatorname{sen} x.$$

$$\frac{dy}{y} = \operatorname{arctg} u \frac{\cos x}{\operatorname{sen} x} dx + L \operatorname{sen} x \frac{du}{1+u^2}$$

$$\therefore dy = \operatorname{sen} x) \operatorname{arctg} u \left(\operatorname{arctg} u \cdot \cot x dx + \frac{L \operatorname{sen} x du}{1+u^2} \right)$$

DIFERENCIAL DE ARC COT u . $y = \operatorname{arc} \cot u$.

$$u = \cot y, \quad du = -\operatorname{cosec}^2 y dy = -(1 + \cot^2 y) dy$$

$$\therefore d \operatorname{arc} \cot u = -\frac{du}{1+u^2}$$

Las diferenciales de $\operatorname{arc} \operatorname{tg} y$ y $\operatorname{arc} \cot$ son iguales con signos contrarios.

DIFERENCIAL DE ARC SEC u . $y = \operatorname{arc} \sec u$

$$u = \sec y \quad du = \operatorname{tg} y \sec y dy = \sqrt{\sec^2 y - 1} \sec y dy$$

$$\therefore d \operatorname{arc} \sec u = \frac{du}{u \sqrt{u^2 - 1}}$$

DIFERENCIAL DE ARC COSEC u . $y = \text{arc cosec } u$

$$u = \text{cosec } y, du = -\cot y \text{ cosec } y dy = -u \sqrt{1-u^2} dy$$

$$\therefore d \text{ arc cosec } u = -\frac{du}{u\sqrt{u^2-1}}$$

DIFERENCIA DE ARC VERSENO. $y = \text{arc vers } u$

$$u = \text{vers } y; du = \text{sen } y dy = \sqrt{2u-u^2} dy$$

$$\therefore d \text{ arc vers } u = \frac{du}{\sqrt{2u-u^2}}$$

DIFERENCIAL DE ARC COVERSENO. $y = \text{arc covers } u$

$$u = \text{covers } y, du = -\cos y dy = -\sqrt{2u-u^2} dy$$

$$\therefore d \text{ arc covers } u = -\frac{du}{\sqrt{2u-u^2}}$$

Ejer. 291. $y = \text{arc sen } \frac{x}{\sqrt{1+x^2}}$ (Bowser 55).

$$dy = d \frac{x}{\sqrt{1+x^2}} : \sqrt{1-\frac{x^2}{1+x^2}} = \frac{1}{\sqrt{(1+x^2)^3}} : \frac{1}{\sqrt{1+x^2}} = \frac{dx}{1+x^2}$$

$$2. y = \text{arc cos } \sqrt{1-x^2}. dy = -\frac{d \sqrt{1-x^2}}{\sqrt{1-(1-x^2)}} = \frac{dx}{\sqrt{1-x^2}}.$$

En efecto, $\cos y = \sqrt{1-x^2} \therefore 1-\cos^2 y = x^2, \therefore y = \text{arc sen } x$.

3. $y = \text{arc sec } \frac{1}{2x^2}$ (Todhunter, 55)

$$d \operatorname{arc} \sec u = \frac{du}{u\sqrt{u^2-1}} \therefore du = -\frac{4x dx}{(2x^2-1)^2},$$

$$\sqrt{u^2-1} = \sqrt{\left(\frac{1}{2x^2-1}\right)^2-1} = \sqrt{\frac{1-(2x^2-1)^2}{(2x^2-1)^2}} = \frac{2x\sqrt{x^2+1}}{2x^2-1}$$

$$dy = -\frac{4x dx}{(2x^2-1)^2} : \frac{2x\sqrt{x^2+1}}{(2x^2-1)^2} = -\frac{2}{\sqrt{1-x^2}} dx$$

$$4. \quad y = \operatorname{tg} x \operatorname{arc} \operatorname{tg} x \therefore \frac{dy}{dx} = \frac{\operatorname{tg} x}{1+x^2} + \operatorname{arc} \operatorname{tg} x \sec^2 x.$$

$$5. \quad y = \mathbf{L} (\operatorname{arc} \cos \sqrt{1+x^2}) \quad (\text{Sturm, 68})$$

$$y = \mathbf{L} \operatorname{arc} \operatorname{sen} x = \frac{dx}{\sqrt{1-x^2} \operatorname{arc} \operatorname{sen} x}$$

$$6. \quad D x e^{\operatorname{arc} \operatorname{tg} x} = e^{\operatorname{arc} \operatorname{tg} x} d \operatorname{arc} \operatorname{tg} x = \frac{e^{\operatorname{arc} \operatorname{tg} x}}{1+x^2} dx$$

$$7. \quad d \operatorname{sen} \operatorname{arc} \operatorname{sen} x = \frac{\cos \operatorname{arc} \operatorname{sen} x}{\sqrt{1-x^2}} dx$$

$$8. \quad d \operatorname{arc} \operatorname{sen} \sqrt{\operatorname{sen} x} = \frac{d\sqrt{\operatorname{sen} x}}{\sqrt{1-\operatorname{sen} x}} \cdot \frac{\cos x}{2\sqrt{\operatorname{sen} x-\operatorname{sen}^2 x}} dx$$

$$= \frac{1}{2} \sqrt{\frac{1-\operatorname{sen}^2 x}{\operatorname{sen} x (1-\operatorname{sen} x)}} dx = \frac{1}{2} \sqrt{\frac{1+\operatorname{sen} x}{\operatorname{sen} x}} dx = \frac{1}{2} \sqrt{1+\operatorname{cosec} x} dx$$

$$9. \quad d \operatorname{arc}^n \operatorname{sen} \operatorname{arc}^m \cos x$$

$$= - \operatorname{arc}^n \operatorname{sen} x \cdot m \operatorname{ar} x^{m-1} \cos x \frac{dx}{\sqrt{1-x}} \\ + \operatorname{arc}^m \cos x \cdot n \operatorname{arc}^{n-1} \operatorname{sen} x \frac{dx}{\sqrt{1-x^2}} \\ = \frac{\operatorname{arc}^{n-1} \operatorname{sen} x \operatorname{arc} \cos^{m-1} x}{\sqrt{1-x^2}} (u \operatorname{arc}^m \cos x \cdot m \operatorname{arc}^n \operatorname{sen} x) dx.$$

$$* 300. \quad u = \operatorname{arc} \operatorname{tg} \frac{5x - 10x^3 + x^5}{1 - 10x^2 + 5x^4} \quad (\text{PEACOCK, } 39^{\circ})$$

$$du = d \operatorname{arc} \operatorname{tg} \frac{v}{t} = \frac{u \frac{v}{t}}{1 + \left(\frac{v}{t}\right)^2} = \frac{tdv - vdt}{v^2 + t^2}$$

$$tdv = 5(1 - 10x^2 + 5x^4)(1 - 6x + x^4)$$

$$vdt = 20x^2(x^2 - 1)(5 - 10x^2 + x^4)$$

$$\therefore du = \frac{5}{1 + x^2} dx$$

30. Formulario diferencial.

FUNCIONES ALGEBRAICAS

- | | |
|------------------------|-------------------------|
| 1. Término constante | $d(a) = 0$ |
| 2. Función logarítmica | $d(L u) = \frac{du}{u}$ |
| 3. Suma de funciones | $d(u+v) = du + dv$ |
| 4. Signo y coeficiente | $d(\pm au) = \pm adu$ |

5. Potencia	$d(u^n) = nu^{n-1} du$
6. Inversa	$d\left(\frac{1}{u}\right) = -\frac{du}{u^2}$
7. Raíz cuadrada	$d(\sqrt{u}) = \frac{du}{2\sqrt{u}}$
8. Producto de funciones	$d(uv) = u dv + v du$
9. Cuociente de funciones	$d\left(\frac{u}{v}\right) = \frac{v du - u dv}{v^2}$

FUNCIONES TRASCENDENTES

10. Función exponencial	$d(a^u) = a^u L a du$
11. » »	$d(e^x) = e^x dx$
12. » »	$d(\text{sen } u) = \cos u du$
13. » coseno	$d(\cos u) = -\text{sen } u du$
14. » tangente	$d(\text{tg } u) = \sec^2 u du$
15. » arco seno	$d(\text{arc sen } u) = \frac{du}{\sqrt{1-u^2}}$
16. » arco tangente	$d(\text{arc tg } u) = \frac{du}{1+u^2}$

31. EJERCICIOS. PRIMERA SERIE.

$$301. y = \frac{e^x - e^{-x}}{e^x + e^{-x}}. \quad (\text{CARR, 263})$$

Cuociente de funciones exponenciales:

$$\begin{aligned} dy &= \frac{(e^x + e^{-x})(e^x + e^{-x}) - (e^x - e^{-x})(e^x - e^{-x})}{(e^x + e^{-x})^2} dx \\ &= \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{(e^x + e^{-x})^2} dx = \frac{4}{(e^x + e^{-x})^2} dx \end{aligned}$$

$$2. y = ab^x = ab^{x^x} \quad (\text{STURM, 59})$$

Función excepcional. Hacemos $u = x^x$ y aplicamos logaritmos:

$$Ly = La + uLb \dots dy = Lbdu.$$

Aplicamos de nuevo L, $Lu = xLx$

$$\dots \frac{du}{u} = \frac{dx}{x} + Lx dx \dots du = x^x(1 + Lx) dx$$

$$\dots dy = ab^x x^x (1 + Lx) dx.$$

$$3. \quad u = L\{L[\dots L(Lx)]\} = L_n x. \quad (\text{GRÉGORY, 2})$$

Sabemos que $dLu = \frac{du}{u}$; y si $u = Lx$:

$$Ld(Lx) = dL_2 x = \frac{dLx}{Lx} = \frac{dx}{xLd}.$$

Sea, ahora, $u = dL_2 x$:

$$dL(L_2 x) = dL_3 x = \frac{dL_2 x}{L_2 x} = \frac{dx}{xLxL_2 x}.$$

Sea $u = L_3 x$:

$$dL(L_3 x) = dL_4 x = \frac{dL_3 x}{L_3 x} = \frac{dx}{xLxL_2 xL_3 x}$$

$$\dots dL^n x = \frac{dx}{xLxL_2 x \dots L_{n-1} x}.$$

$$4. \quad d(Ae^{mx} + Be^{-mx}) \quad (\text{COMBEROUSSE, 490})$$

$$= (Ae^{mx} \cdot m - Be^{-mx} m) dx = m(Ae^{mx} - Be^{-mx}) dx$$

$$\begin{aligned}
 5. \quad de^{e^x} &= de^u = e^u de^x = e^u dv = e^u e_v dx \\
 &= e^{e^x} e^x dx
 \end{aligned}$$

$$\begin{aligned}
 6. \quad dL \frac{a+bx}{a-bx} &= d[L(a+bx) - L(a-bx)] \\
 &= \left(\frac{b}{a+bx} + \frac{b}{a-bx} \right) dx = \frac{2ab}{a^2 - b^2 x^2} dx.
 \end{aligned}$$

$$\begin{aligned}
 7. \quad dL \sqrt{\frac{1+x}{1-x}} &= \frac{1}{2} d[L(1-x) - L(1+x)] \\
 &= \frac{1}{2} \left(-\frac{1}{1-x} - \frac{1}{1+x} \right) dx = \frac{dx}{x^2 - 1}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad dL[(x-a)^m (x-b)^n (x-c)^p \dots] \\
 &= d[mL(x-a) + nL(x-b) + pL(x-c) \dots] \\
 &= \left(\frac{xm}{x-a} + \frac{n}{x-b} + \frac{p}{x-c} + \dots \right) dx
 \end{aligned}$$

$$9. \quad dL(x + \sqrt{x^2 - a^2}) + \operatorname{arc} \sec \frac{x}{a} \quad (\text{Id. 511})$$

$$\begin{aligned}
 dy &= \frac{d(x + \sqrt{x^2 - a^2})}{x + \sqrt{x^2 - a^2}} + d \frac{x}{a} : \frac{x}{a} \sqrt{\frac{x^2}{a^2} - 1} \\
 &= \left[\frac{\sqrt{x^2 - a^2} + x}{(x + \sqrt{x^2 - a^2}) \sqrt{x^2 - a^2}} + \frac{a}{x \sqrt{x^2 - a^2}} \right] dx = \frac{1}{x} \sqrt{\frac{x+a}{x-a}}
 \end{aligned}$$

$$\begin{aligned}
 310. \quad d \operatorname{arc} \operatorname{tg} \frac{a+x}{1-ax} &= d \frac{a+x}{1-ax} : \left[1 + \left(\frac{a+x}{1-ax} \right)^2 \right] \\
 &= \frac{(1-ax) + (a+x)a}{(1-ax)^2} \cdot \frac{(1-ax)^2}{(1-ax)^2 + (a+x)^2} dx = \frac{1}{1+x^2} ax
 \end{aligned}$$

(Continuará)

